8th Alexander Friedmann International Seminar
on Gravitation and Cosmology
International Journal of Modern Physics: Conference Series
Vol. 3 (2011) 396–407

DOI: 10.1142/S2010194511001498

AN ANALYSIS OF A REGULAR BLACK HOLE INTERIOR

DANIELA PÉREZ

Instituto Argentino de Radioastronomía, Camino Gral Belgrano Km 40
C.C.5, (1984) Villa Elisa, Bs. As., Argentina
danielaperez@iar.unlp.edu.ar

GUSTAVO E. ROMERO

Instituto Argentino de Radioastronomía, Camino Gral Belgrano Km 40
C.C.5, (1984) Villa Elisa, Bs. As., Argentina
and
FCAG, Universidad de La Plata, Paseo del Bosque s/n
(B1900FWA) La Plata, Buenos Aires, Argentina
romero@iar.unlp.edu.ar

CAMILA A. CORREA

Facultad de Ciencias Astronómicas y Geofísicas, UNLP, Paseo del Bosque s/n
CP (1900), La Plata, Bs. As., Argentina
camilacorrea@carina.fcaglp.unlp.edu.ar

SANTIAGO E. PEREZ BERGLIAFFA

Departamento de Fsica Terica, Instituto de Fsica, Universidade do Estado do Rio de Janeiro,
Rua So Francisco Xavier 524, Maracan Rio de Janeiro - RJ, Brasil, CEP: 20550-900
sepbergliaffa@gmail.com

Received 10 June 2011
Revised 7 July 2011

In this work we address the issue of the thermodynamic behavior of a non-singular spherically-symmetric black hole model introduced by Mbonye and Kanzanas (2005). This spacetime is described by Schwarschild's solution at large radius and by a de Sitter-like solution at small radius. The interior of the black hole consists of matter fields with sound speed bounded by the speed of light. The matter transits smoothly between normal matter and a core of an exotic fluid with an equation of state that approaches $p = -\rho$ when $r \to 0$. We derive the general equations of the thermodynamic quantities for an arbitrary matter density profile, and adjust the results to the specific regular black hole. We also calculate the Weyl and Kretschmann scalars and analyse the behaviour of the gravitational field in connection with the thermodynamics of the matter fields. Finally, we study whether the regular behaviour at the center affects the thermodynamics of the black hole, and we discuss a possible physical interpretation of the state of regular black hole interiors.

Keywords: Black hole; gravitation; singularity.

PACS numbers: 04.70.-s, 95.30.Sf, 04.20.Dw

1. Introduction

A special type of regular black hole is characterized by the presence of matter at the center of a static configuration with an equation of state of the form $p = -\rho$. The repulsive behaviour of the matter prevents complete gravitational collapse. Hence, the description of the spacetime geometry is singularity-free.

The occurrence of spacetime singularities, implied by the singularity theorems,[1−3] is usually understood as a breakdown in the descriptive power of General Relativity. It is usually thought that only by means of a quantum theory of gravity singularities could be removed. Singularity theorems posit certain energy conditions for the collapsing matter. These assumptions, however, might not longer hold for matter under extreme conditions, whose properties are totally unknown.

Sakharov gave the first insights of a nonsingular collapse considering the equation of state $p = -\rho$ for a superdense fluid,[4] and Bardeen suggested that such a fluid could be the final state of gravitational collapse.[5] An exact solution of the Einstein field equations describing a nonsingular black hole containing at the core a de Sitter fluid was found by Dymnikova.[6] The main problem, however, with many models of regular black holes is that they do not provide an transition between the different types of matter in the black hole interior appropriate

Mbonye and Kazanas[7] made a recent attempt to construct a model using an explicit equation of state that smoothly matches a de Sitter interior to a Schwarzschild spacetime geometry. The main goal of the present work is to study the thermodynamic properties of the complete internal black hole structure of such a spacetime.

2. A Model of Regular Black Hole

A static spherically symmetric geometry can be described by the line element:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{1}$$

In order to obtain the line element that describes a regular black hole, it is necessary to calculate the expressions for $B(r)$ and $A(r)$. We adopt the equation of state suggested by Mbonye and Kazanas:[7]

$$p_{\mathsf{r}} = \left[\alpha - (\alpha+1)\left(\frac{\rho}{\rho_{\mathsf{max}}}\right)^2\right]\left(\frac{\rho}{\rho_{\mathsf{max}}}\right)\rho, \tag{2}$$

with $\alpha = 2.2135$. The value of α was chosen in order to have finite sound speed, bounded by the speed of light. The physical state changes smoothly between normal matter and a core of an "exotic fluid" with an equation of state that approaches $p = -\rho$ when $r \to 0$. The plot of Eq. (2) is shown in Fig. 1.

The solution to the Einstein field equations, $G^{\mu}_{\nu} = 8\pi T^{\mu}_{\nu}$, for spherical symmetry and the equation of state (2) takes the form:[8]

$$ds^2 = -B(r)dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3}$$

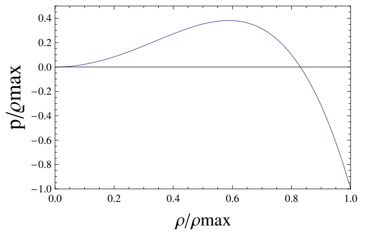

Fig. 1. Equation of state of the matter fields.

with,

$$B(r) = \exp\left\{ \int_{r_0}^{r} \frac{2}{r'^2} \left[m(r') + 4\pi r'^3 p_{\mathsf{r}'} \right] \left[\frac{1}{\left(1 - \frac{2m(r')}{r'}\right)} \right] dr' \right\}, \tag{4}$$

where the mass $m(r)$ is given by $m(r) = 4\pi \int_0^r \rho(r') r'^2 dr'$. In this model the fluid is anisotropic and the energy-momentum tensor is such that $T_2^2 = T_3^3 \neq T_1^1$ and $T_1^1 \neq T_0^0 \neq T_2^2$.

Equation (3) describes the geometry of the spacetime of a regular black hole with matter and a de Sitter core. We can derive from the g_{rr} component of the metric that this spacetime presents an event horizon at $R_{\mathrm{S}}=2M$, in units of $c = G = 1$; M stands for the mass of the regular black hole.

By solving the complete system of Einstein field equations, it is obtained the expression for the tangential pressure:

$$p_{\perp} = p_{\mathsf{r}} + \frac{r}{2} p_{\mathsf{r}}' + \frac{1}{2} \left(p_{\mathsf{r}} + \rho \right) \left[\frac{m(r) + 4\pi r^3 p_{\mathsf{r}}}{r - 2m(r)} \right], \tag{5}$$

where p_{r} is given by Eq. (2). The formula of the tangential pressure is a generalization of the Tolman-Oppenheimer-Volkoff equation.

In order to study the thermodynamical properties of this spacetime we work with the normalization of the density profile suggested by Dymnikova:[9]

$$\rho(r) = \rho_{\mathsf{max}} e^{-8\frac{r^3}{R^3}}, \tag{6}$$

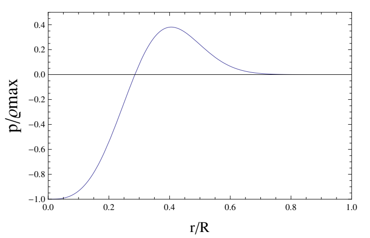

Fig. 2. Pressure as a function of radial coordinate inside the regular black hole considered in the text.

where, $R^3 = 8R_{\rm S}r_0^2$ with:

$$r_0 = \left(\frac{3}{8\pi\rho_{\mathsf{max}}}\right)^{1/2}, \tag{7}$$

and $\rho_{\rm max}$ is of the order of the Planck density. Notice that all the mass is contained inside a sphere of radius R within the black hole. Outside this radius there is empty spacetime.

The plot of Eq. (2) as a function of the radial coordinate is shown in Fig. 2. It can be seen that the pressure follows the equation $p = -\rho$ at the core of the object, and it goes to zero at $r/R = 1$, the surface of the matter region. The pressure is also zero at $r/R = 0.28$; for $r/R > 0.28$ the pressure is positive and for $r/R < 0.28$ it is negative. The exotic matter field extends from $r/R = 0.4$ inwards. The function p_r has one absolute maximum at $r/R = 0.4$ and two inflexion points at $r/R = 0.26$ and $r/R = 0.5$.

3. Thermodynamics of the Matter Fields

In this work we assume that the collapsed body has reached a static equilibrium configuration.[7] Matter is also assumed to be in thermodynamic equilibrium. In the black hole interior the attractive force of the gravitational field is balanced by the repulsion exerted by the exotic matter. Since we are considering a system in equilibrium, the thermodynamic quantities can be calculated from the laws of standard thermodynamics. In particular, the temperature of the matter as a function of the radius can be computed from:

$$TdS = d(\rho V) + pdV. \tag{8}$$

From Eq. (8):

$$dp = \frac{\rho + p}{T} dT. \tag{9}$$

Then, replacing Eqs. (2) and (6) into (9), we obtain:

$$\frac{T}{T_{\mathsf{sup}}} = \left[1 + \alpha e^{-8r^3/R^3} - (\alpha+1)e^{-24r^3/R^3}\right]^{4/3} e^{\Xi(r)}, \tag{10}$$

where $T_{\rm sup}$ stands for the temperature of the the matter field at $r = R$, and:

$$\Xi(r) = \frac{2}{3}\int_0^{e^{-8r^3/R^3}} \frac{\alpha d(\rho/\rho_{\mathsf{max}})}{1 + \alpha(\rho/\rho_{\mathsf{max}}) - (\alpha+1)(\rho/\rho_{\mathsf{max}})^3}. \tag{11}$$

In Fig. 3 we offer the plot of the temperature of the matter as a function of the radius. We note that the temperature tends to absolute zero close to the core. As for the pressure, there is only one maximum at the same radial coordinate, $r/R = 0.4$. As expected, higher temperatures correspond to higher average kinetic energy of the particles, and hence to higher pressure. The pressure of a fluid of mixed mass and fixed volume is directly proportional to the fluid's absolute temperature. From

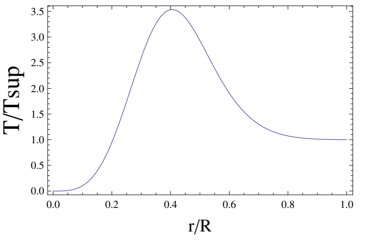

Fig. 3. Temperature as a function of radial coordinate inside the black hole. T_{sup} stands for the temperature of the the matter field at $r = R$.

Eq. (10) we calculate the inflexion points of $T(r)$; they occur at $r/R = 0.31$ and at $r/R = 0.57$.

We obtain an expression for the entropy up to an additive constant substituting (9) in (8):

$$S \equiv \frac{(\rho + p)}{T} V. \tag{12}$$

It is plotted in Fig. 4. The entropy goes to zero as $r \to 0$ in accordance to Nerst's theorem. There is a clear maximum close to $r/R = 0.4$, in the region of highest density of normal matter.

We can also estimate the entropy density of the matter inside the regular black hole, obtaining:

$$\frac{s}{s_{\mathsf{R}/2}} = \frac{(\rho/\rho_{\mathsf{max}})[1 + \alpha(\rho/\rho_{\mathsf{max}}) - (\alpha+1)(\rho/\rho_{\mathsf{max}})^3]^{-1/3}}{0.2076 e^{(2/3)\int_0^\rho \alpha d\rho [\rho_{\mathsf{max}} + \alpha\rho - (\alpha+1)(\rho^3/\rho^2_{\mathsf{max}})]^{-1}}}. \tag{13}$$

The plot of the entropy density as a function of r and ρ is shown in Figs. 5 and 6. It can be seen that the entropy density diverges at the origin as a consequence of the vanishing volume. The entropy density as a function of the density has one inflexion point at $\rho/\rho_{\mathrm{max}} = 0.73$ which is located at $r/R = 0.34$.

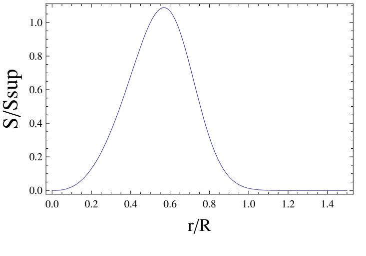

Fig. 4. Entropy of the matter field as a function of radial coordinate inside the black hole. S_{sup} stands for the entropy of the the matter field at $r = R$.

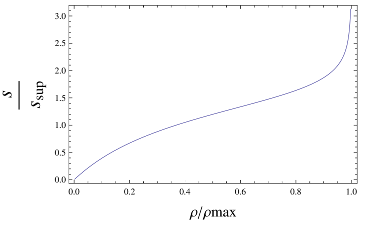

Fig. 5. Entropy density of the matter inside the black hole as a function of the density. S_{sup} stands for the entropy of the matter field at $r = R$.

In order to calculate the speed of sound as a function of the energy density, we solve $(v/c)^2 = dp/d\rho$:

$$\left(\frac{v}{c}\right)^2 = 2e^{-8r^3/R^3}\left[\alpha - 2(\alpha+1)e^{-16r^3/R^3}\right]. \tag{14}$$

The result is shown in Fig. 7. From Figs. 2 and 7, we see that the sound speed is zero for the same value of r/R at which the pressure is maximum. In addition, the speed of sound takes complex values in the region $r/R < 0.4$ (i.e. sound waves do not exist). This can be derived from the negative slope of the equation of state as a function of the density [see Eq. (2) and Fig. 1]. The sound waves cannot propagate in the region $r/R < 0.4$ because a variation in pressure causes an expansion rather than a compression in the fluid. Because of the repulsive behaviour of the matter, this region is opaque to sound waves. Furthermore, it seems that that the amplitude of sound waves has an exponential modulation as they get close to $r/R = 0.4$.

The specific heat at constant volume as a function of the radial coordinate is:

$$C_V = T\left(\frac{dS}{dT}\right)_V. \tag{15}$$

Considering the equations obtained previously:

$$C_V = TV\left(\frac{d\rho + dp}{dT} - \frac{\rho + p}{T}\right)_V, \tag{16}$$

$$C_V = V\frac{d\rho}{dT}. \tag{17}$$

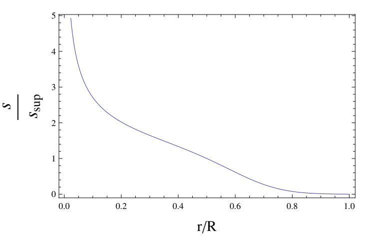

Fig. 6. Entropy density of the matter inside the black hole as a function of radial coordinate. S_{sup} stands for the entropy of the matter field at $r = R$.

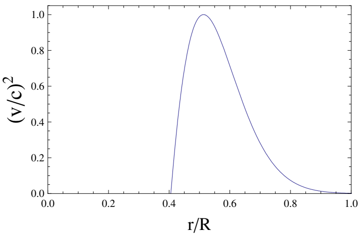

Fig. 7. Sound speed as a function of radial coordinate in the regular black hole interior.

We calculate the final expression of C_V by introducing the expression of temperature as a function of the energy density and changing variables:

$$\frac{dT}{T_{\text{sup}}} = \frac{2[\alpha - 2(\alpha+1)(\rho/\rho_{\max})^2]d\rho}{\rho_{\max} + [\alpha - (\alpha+1)(\rho/\rho_{\max})^2]\rho}, \tag{18}$$

$$\frac{C_V}{C_{V\text{sup}}} = 2\alpha\frac{(1 + [\alpha - (\alpha+1)(\rho/\rho_{\max})^2](\rho/\rho_{\max})}{\alpha - 2(\alpha+1)(\rho/\rho_{\max})^2}, \tag{19}$$

$$\frac{C_V}{C_{V\text{sup}}} = 2\alpha\frac{1 + [\alpha - (\alpha+1)e^{-16r^3/R^3}]e^{-8r^3/R^3}}{0.4521(\alpha - 2(\alpha+1)e^{-16r^3/R^3})}, \tag{20}$$

where $C_{V\text{sup}}$ denotes the specific heat at constant volume at $r = R$. The plot of the specific heat at constant volume is shown in Fig. 8.

The specific heat at constant pressure is calculated from:

$$C_P = T\left(\frac{dS}{dT}\right)_P, \tag{21}$$

$$C_P = V\frac{d\rho}{dT} + (\rho + p)\frac{dV}{dT} - \frac{(\rho+p)V}{T}, \tag{22}$$

$$C_P/V = \frac{d\rho}{dT} + (\rho + p)\frac{d\ln V}{dT}. \tag{23}$$

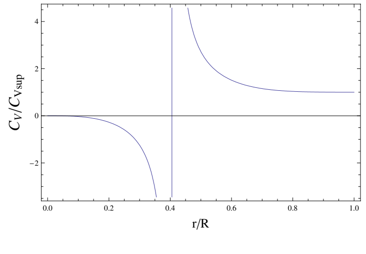

Fig. 8. Specific heat at constant volume as a function of radial coordinate in the black hole interior. $C_{V\text{sup}}$ stands for the specific heat at constant volume at $r = R$.

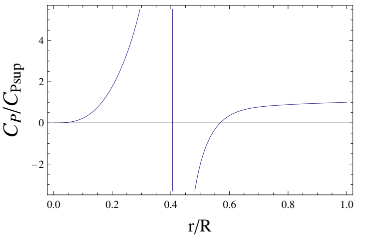

Fig. 9. Specific heat at constant pressure as a function of radial coordinate in the black hole interior. $C_{P\mathrm{sup}}$ stands for the specific heat at constant pressure at $r = R$.

Since:

$$V = 4/3\pi r^3, \tag{24}$$

we obtain:

$$\begin{aligned}\frac{C_P}{C_{P\mathrm{sup}}} = &\left(1 - \frac{1}{8r^3}(1 + [\alpha - (\alpha+1)e^{-16r^3/R^3}]e^{-8r^3/R^3})\right)\\ &\times \frac{\left(1 + [\alpha - (\alpha+1)e^{-16r^3/R^3}]e^{-8r^3/R^3}\right)}{0.395(\alpha - 2(\alpha+1)e^{-16r^3/R^3})},\end{aligned} \tag{25}$$

where $C_{P\mathrm{sup}}$ stands for the specific heat at constant pressure at $r = R$. In Fig. 9 we show the plot of the specific heat at constant pressure.

We find that both the specific heat at constant volume and constant pressure are not defined for $r/R = 0.4$, where there is the change from normal to exotic matter. The specific heat C_V is negative for $r/R < 0.4$ and it is positive for $r/R > 0.4$. This behaviour can be understood in terms of Eq. (17), the regions where the temperature increases and decreases, and the density profile given in Eq. (6). In Fig. 9 we can see that $C_P = 0$ at $r/R = 0.57$; this point coincides with one of the inflexion points of the temperature. C_P is positive for $r/R < 0.4$ and for $r/R > 0.57$. In the region $0.4 < r/R < 0.57$, C_P is negative. These results show that the specific heats are not defined for the same value of r/R where the sound speed equals to zero. This suggests a possible region of instability for the normal matter field: sound waves should bounce at $r/R = 0.4$ and might accumulate energy in the region between this boundary and the horizon.

4. Behaviour of the Gravitational Field

The Weyl tensor is a 4-rank tensor that contains the independent components of the Riemann tensor not captured by the Ricci tensor. It can be considered as the traceless part of the Riemann tensor. The Weyl tensor is given by:

$$\begin{aligned}C_{\alpha\beta\gamma\delta} = &R_{\alpha\beta\gamma\delta} + \frac{2}{n-2}(g_{\alpha[\gamma}R_{\delta]\beta} - g_{\beta[\gamma}R_{\delta]\alpha})\\ &+ \frac{2}{(n-1)(n-2)}R\, g_{\alpha[\gamma}g_{\delta]\beta},\end{aligned} \tag{26}$$

where $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor, $R_{\alpha\beta}$ is the Ricci tensor, R is the Ricci scalar, [ ] refers to the antisymmetric part, and n is the number of dimensions of spacetime.

The absence of structure in spacetime corresponds to the absence of Weyl conformal curvature ($W = C^{abcd}C_{abcd} = 0$). Hence, the Weyl tensor contains the information of the gravitational field even in absence of matter and other non-gravitational fields.

The Weyl scalar for the spacetime metric given in Eq. (3) is:

$$\begin{aligned} W = {} & \frac{4}{3}\pi^2\rho_{\mathtt{max}}^2(p_8\,x^8 + p_6\,x^6 + p_5\,x^5 + p_4\,x^4 + p_3\,x^3 + p_2\,x^2 \\ & + p_1\,x + p_0)^2 / \left[\left(3r - \pi R^3\rho_{\max} + \pi R^3\rho_{\max}\,x\right) r^6 R^6\right], \end{aligned} \tag{27}$$

where $x = e^{-8\frac{r^3}{R^3}}$ and the coefficients p_{i} are:

$$\begin{aligned} p_0 &= 3R^6 r - R^9\pi\rho_{\max}, \\ p_1 &= 2R^9\pi\rho_{\max} - 3R^6 r + 6r^3R^6\pi\rho_{\max} - 24r^4R^3, \\ p_2 &= -6r^3R^6\pi\rho_{\max} - R^9\pi\rho_{\max} + 576r^7\alpha - 192r^6\alpha\pi R^3\rho_{\max} - 2R^6r^3\alpha\pi\rho_{\max}, \\ p_3 &= 2R^6r^3\alpha\pi\rho_{\max} + 144r^6\alpha\pi R^3\rho_{\max}, \\ p_4 &= (-1152 - 1152\alpha)r^7 - 48\pi R^3\rho_{\max}(-8\alpha - 8 + \alpha^2)r^6 + \\ &\quad + 2\pi\rho_{\max}R^6(\alpha+1)r^3, \\ p_5 &= -336\pi\rho_{\max}R^3(\alpha+1)r^6 - 2\pi\rho_{\max}R^6(\alpha+1)r^3, \\ p_6 &= 96\pi\rho_{\max}\alpha R^3(\alpha+1)r^6, \\ p_8 &= -48\pi R^3\rho_{\max}(\alpha+1)^2r^6. \end{aligned} \tag{28}$$

If we let $r \to 0$, the Weyl scalar goes to zero. Since the regular black hole spacetime has a de Sitter geometry in the origin, this is the correct limit.

The plot of the Weyl scalar is shown in Fig. 10. We can see that for large values of r the Weyl scalar tends asymptotically to zero. In the matter region ($r/R < 1$) it has one absolute maximum at $r/R = 0.5$ and one relative maximum at $r/R = 0.26$. Between these points, $W = 0$ at $r/R = 0.34$. If we analyse the equation of state as a function of the density (2), the only inflexion point occurs at $r/R = 0.5$. The absolute maximum of the Weyl scalar seems to be related to the transition point of different types of matter of two regions.

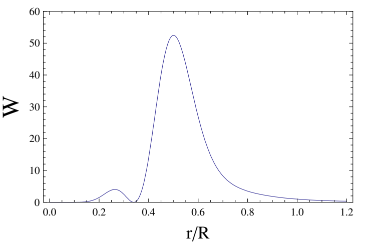

Fig. 10. Weyl scalar as a function of radial coordinate.

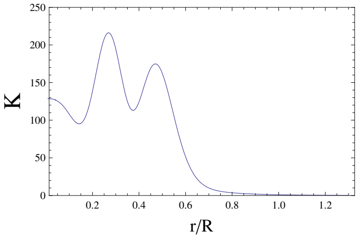

Fig. 11. Kretschmann scalar as a function of radial coordinate.

The value of r/R for which the Weyl scalar has one relative maximum is close to the point where the pressure is zero. This suggests a relation between the region where matter has a negative pressure and the behaviour of the Weyl scalar. Furthermore, the inflexion point of the entropy density of the matter coincides with the value of r/R for which the Weyl scalar is zero, that is at $r/R = 0.34$. At this point the gravitational field transits from attractive to repulsive.

We also calculate the Kretschmann scalar. The plot is shown in Fig. 11. Again, we can see that outside the matter region the Kretschmann scalar tends asymptotically to zero. If we let $r \to 0$, it goes to:

$$K = \frac{512\pi^2 {\rho_{\max}}^2}{3}, \tag{29}$$

which is the expected value for the de Sitter spacetime. The Kretschmann scalar is always positive and it has one absolute maximum at $r/R = 0.269$ and one relative maximum at $r/R = 0.47$. Furthermore, the value of the coordinate r for which the curvature of the spacetime is maximum corresponds to the region of $p = 0$.

The Weyl and Kretschmann scalar verify the following relation:

$$R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} \geq C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta}, \tag{30}$$

which holds in every static spherically symmetric spacetimes.[10]

Penrose[11] suggested that entropy might be assigned to the gravitational field itself and he proposed that the Weyl curvature tensor can be used to specify it. The behaviour of the Weyl tensor follows what is expected for the gravitational entropy throughout the history of the universe: it is zero in the (homogeneous) Friedmann-Robertson-Walker model and it is large in the (maximally inhomogeneous) Schwarzschild spacetime.

Rudjord, Gr∅n and Sigbj∅rn[10] made a recent attempt to develop a description of the gravitational entropy based on the construction of a scalar derived from the contraction of the Weyl tensor and the Riemann tensor:

$$P^2 = \frac{C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta}}{R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta}} = \frac{W}{K} \tag{31}$$

This approach is based on matching their description of the entropy of a black hole in the event horizon with the Hawking-Bekenstein entropy.[12] In particular, they

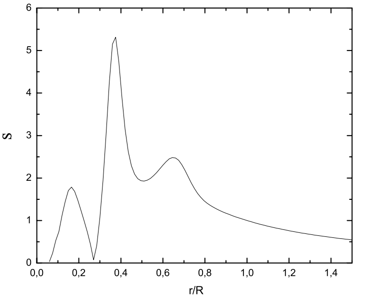

Fig. 12. Gravitational entropy density as a function of radial coordinate.

calculated the entropy of Schwarzschild black holes and the Schwarzschild-de-Sitter spacetime.

They describe the gravitational entropy of a black hole by the surface integral:

$$S = k_{\mathrm{s}} \int_{\sigma} \vec{\Psi} \cdot d\vec{\sigma}, \tag{32}$$

where σ is the horizon of the black hole and the vector field $\vec{\Psi}$ is:

$$\vec{\Psi} = P\vec{e}_r. \tag{33}$$

Rudjord et al. required Eq. (32) to be equal at the horizon to the Hawking-Bekenstein entropy:

$$S = S_{\mathrm{HB}}. \tag{34}$$

The latter equation allows to calculate the constant k_{s}.

Finally, the entropy density can be determined by means of Gauss's divergence theorem, rewriting Eq. (32) as a volume integral:

$$\mathfrak{s} = k_{\mathrm{s}} \mid \nabla \cdot \vec{\Psi} \mid, \tag{35}$$

where the absolute value brackets were added to avoid negative values of entropy.

We compute the gravitational entropy density following Rudjord et. al. proposal. The plot of the entropy density is shown in Fig. 12. It can be seen that $\mathfrak{s} = 0$ at $r/R = 0.27$, which is close to the transition point from the regular to the exotic matter regions. The absolute maximum is at $r/R = 0.37$ and the two relative maxima are at $r/R = 0.16$ and at $r/R = 0.65$. For large values of r/R the gravitational entropy density tends asymptotically to zero and in the core of the object it is also zero. The latter result is in accordance to what it is expected for a good description of the entropy of the gravitational field. We remark that the gravitational entropy on the horizon is equal to the Bekenstein-Hawking entropy, and can be measured by external observers (see the paper by Rudjord et al.[10] for further discussion). On the contrary, the thermodynamical quantities associated with the matter field inside the event horizon can be measured only inside the black hole.

5. Conclusions

We have studied the thermodynamical properties of a regular black hole interior described by an equation of state that provides a smooth transition from regions of normal matter to an exotic fluid with de Sitter-like behaviour. The exotic region supports the object from total collapse leading to an equilibrium configuration for the matter and fields. This allows a thermodynamical analysis of the black hole interior. We have found that sound waves do not penetrate the inner exotic core, where the temperature and the entropy go to zero.

We also studied the behaviour of the gravitational field in the system through the Weyl and Kretschmann tensors. We adopted a well-defined estimator for the gravitational entropy density based on the Weyl scalar and found the expected behaviour for a de Sitter-Schwarzschild spacetime. This reinforces the confidence on this classical estimator of what should be essentially a statistical property of a quantified field.

The fact that the amplitude of sound waves has an exponential modulation as they approach $r/R = 0.4$, where the specific heats are not defined, suggests that exterior perturbations (like accretion of matter) can lead to the accumulation of energy in the region of normal matter. This issue, which can modify the interior through instabilities, is being analysed by a calculation of perturbations that we will discuss in a forthcoming paper.

Acknowledgments

Relativistic astrophysics with G. E. Romero is supported by CONICET through grant PIP 2010/0078 and Spanish MICINN under grant AYA2010-21782-C03-01. S. E. Perez Bergliaffa acknowledges support from UERJ, FAPERJ, and CNPQ.

References

1. R. Penrose, *Phys. Rev. Lett.* **14**, 57 (1965).
2. S. W. Hawking and R. Penrose, *Proc. R. Soc. A* **314**, 529 (1970).
3. S. W. Hawking and G. R. Ellis, *Astrophys. J.* **152**, 25 (1968).
4. A. D. Sakharov, *Sov. Phys. JETP* **22**, 241 (1966).
5. J. M. Bardeen, Non-singular general-relativistic gravitational collapse, in *Proccedings of GR5*, ed. V. A. Fock (Tbilisi University Press, Tbilisi, 1968), p. 174.
6. I. Dymnikova, *Int. J. Mod. Phys. D* **12**, 1015 (2003).
7. M. R. Mbonye and D. Kazanas, *Phys. Rev. D* **72**, 024016 (2005).
8. M .R. Mbonye, N. Battista and B. Farr, *Int. J. Mod. Phys. D* **20**, 1 (2011).
9. I. Dymnikova, *Gen. Rel. Grav.* **24**, 235 (1992).
10. ∅. Rudjord, ∅. Gr∅n and H. Sigbj∅rn, *Phys. Scr.* **77**, 055901 (2008).
11. R. Penrose, Singularity and time-asymmetry, in *General Relativity, an Einstein Centenary Survey*, eds. S.W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, 1979), p. 581.
12. J. D. Bekenstein, *Phys. Rev. D* **9**, 3292 (1974).